UDC 539.3

ON STRESS CONCENTRATION IN A TRANSTROPIC PLATE WITH A CYLINDRICAL CAVITY

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A. S. KOSMODAMIANSKII, I. Kh. MILISHENSKII, and V. A. SHALDYRVAN
(Donetsk)

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The effect of anisotropy of the material on the stress concentration near a cavity weakening a thick plate is studied, using the method given in [1]. Results of numerical computations are given for the case of axisymmetric deformation of cadmium and zinc plates. The stress concentration in an isotropic plate with a cavity, loaded uniformly along the generatrix, was first studied in [2]. Analogous formulation was used in [3] for transtropic plates (for small relative thicknesses) to solve the problem by means of asymptotic methods.

1. We consider the tension-compression problem for a transtropic plate under the following boundary conditions:

$$\sigma_{\xi\zeta} = \sigma_{\eta\zeta} = \sigma_{\zeta\zeta} = 0, \quad \zeta = \pm 1 \tag{1.1}$$

$$\sigma_{rr}|_\Omega = Pg(\zeta), \quad \sigma_{r\zeta}|_\Omega = 0, \quad \sigma_{r\theta} \equiv 0, \quad g(\zeta) = g(-\zeta)$$

$$\xi = x_1 / R, \quad \zeta = x_3 / (\lambda R), \quad \lambda = h / R \tag{1.2}$$

Here R and $2h$ denote the dimensional radius of the cavity and plate thickness, respectively. We construct a solution satisfying the Lamé system and boundary conditions (1.1), in the form of a series in terms of the homogeneous Lur'e–Lekhnitskii solutions. Using the notation of [1] we obtain for the present case,

$$u(r,\zeta) = -\frac{a}{r} + \sum_p n_p(\zeta) P_0^-(\gamma_p^* r)\alpha_p K_0(\gamma_p^* r) \tag{1.3}$$

$$w(r,\zeta) = \sum_p q_p(\zeta)\alpha_p K_0(\gamma_p^* r)$$

Here $(u, 0, w)$ are the components of the displacement vector; σ_{ij} are the components of the stress tensor and a, α_p are arbitrary constants to be determined. The summation is extended over all integral values of p except zero, and the remaining notation follows that of [1].

We assume that the plate material is such that its elastic parameters ν, ν_z, ν_2 and s_0^2 satisfy the conditions

$$b_1 > 0, \quad b_1^2 < b_2, \quad b_1 = \frac{s_0^2 - \nu_2}{1-\nu}, \quad b_2 = \frac{\nu_2}{\nu_z}\frac{1-\nu_2\nu_z}{1-\nu^2}$$

Then the quantities $\gamma_p = \lambda\gamma_p^*$ are roots of the transcendental equation [1]

$$\beta \sin 2\alpha\gamma + \alpha \operatorname{sh} 2\beta\gamma = 0$$

It can be verified directly that in the present case it is sufficient to consider only the roots appearing in the half-plane $\operatorname{Re}\gamma > 0$. The conditions (1.2) which are still to be satisfied, can be written with the help of (1.3) in the form

$$X_0 + 2\mathrm{Re}\sum_p [l_p(\zeta) - n_p(\zeta)P_0^-(\gamma_p^*)]\, X_p - Pg(\zeta) = 0 \qquad (1.5)$$

$$2\mathrm{Re}\sum_p r_p(\zeta)\, P_0^-(\gamma_p^*)\, X_p = 0$$

where the summation is carried out over γ_p from the first quadrant, i.e. the sign of $\mathrm{Re}\,\Sigma$ extends over every positive integral p; the quantities X_0 and X_p are connected with the basic unknowns a and α_p by the relations

$$X_0 = a,\ X_p = \alpha_p K_0(\gamma_p^*)$$

Requiring that the discrepancies in the boundary conditions (1.5) be orthogonal on the interval $[-1, 1]$ to the complete system of functions $\{\sin\delta_m s_0\zeta, \cos\delta_m s_0\zeta\}$, we arrive, after certain formal transformations, at the following infinite system of equations:

$$\mathrm{Re}\sum_p [l_{mp} - n_{mp}P_0^-(\gamma_p^*)]\, X_p = Pg_m \qquad (1.6)$$

$$\mathrm{Re}\sum_p r_{mp}P_0^-(\gamma_p^*)\, X_p = 0, \qquad m = 1, 2, \ldots$$

$$X_0 = Pg_0 - \mathrm{Re}\sum_p [l_{0p} - n_{0p}P_0^-(\gamma_p^*)]\, X_p$$

where

$$l_{mp} = \gamma_p^*\,[(A_{11}\gamma_p^* + s_1S_{1p}A_{13})b_{mp1}\cos\gamma_p s_2 \sin\gamma_p s_1 - s_3(A_{11}\gamma_p^* + s_2S_{2p}A_{13})b_{mp2}\cos\gamma_p s_1\sin\gamma_p s_2]$$

$$n_{mp} = b_{mp1}\cos\gamma_p s_2\sin\gamma_p s_1 - s_3 b_{mp2}\cos\gamma_p s_1 \sin\gamma_p s_2$$

$$r_{mp} = \frac{1}{2s_0^2}\,[(S_{1p} - \gamma_p^* s_1)\,c_{mp1}\cos\gamma_p s_2\sin\gamma_p s_1 - s_3(S_{2p} - s_2\gamma_p^*)\,c_{mp2}\cos\gamma_p s_1\sin\gamma_p s_2]$$

$$g_m = \int_{-1}^{1} g(\zeta)\cos\delta_m s_0\zeta\, d\zeta$$

$$b_{mpj} = \frac{1}{\gamma_p s_j - \delta_m s_0} + \frac{1}{\gamma_p s_j + \delta_m s_0}, \quad c_{mpj} = \frac{1}{\gamma_p s_j - \delta_m s_0} - \frac{1}{\gamma_p s_j + \delta_m s_0}$$

The passage to the limit $\nu_z \to \nu$ and $E_z \to E$ yields a solution of the tension-compression problem for a thick isotropic plate with a cavity which was studied, as we already remarked, in [2].

2. Computations were carried out for cadmium and zinc plates. The elastic constants of these materials are $\nu = 0.054$, $\nu_z = 0.261$, $\nu_2 = 0.939$, $s_0^2 = 2.03$ and $\nu = -0.212$, $\nu_z = 0.272$, $\nu_2 = 1.179$, $s_0^2 = 1.813$ respectively. The loading of the lateral surface Ω was taken in the form $g(\zeta) = \zeta^{2q}$ $(q = 0, 1, \ldots)$. The roots of the characteristic equation (1.4) which were needed for the numerical computations were found using Newton's method with the initial values obtained from the asymptotic formulas. The latter can be written with the help of the results of [5] in the following form:

$$\gamma_p = \frac{1}{2}\,\frac{\beta - i\alpha}{\beta^2 + \alpha^2}\left(\ln\frac{\beta}{\alpha} - \frac{\pi}{2}i + 2ip\pi\right)$$

Table 1, below gives the first ten roots for the materials used (the first two columns for cadmium, $\alpha = 1.184$, $\beta = 0.498$.; the last two for zinc, $\alpha = 1.067$, $\beta = 0.785$).

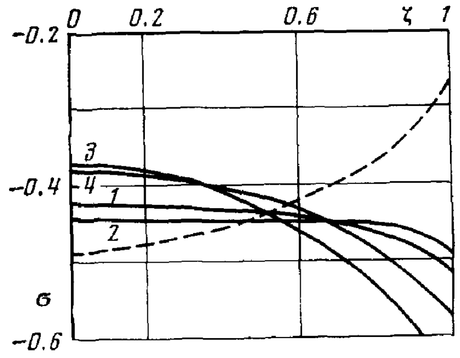

Fig 1

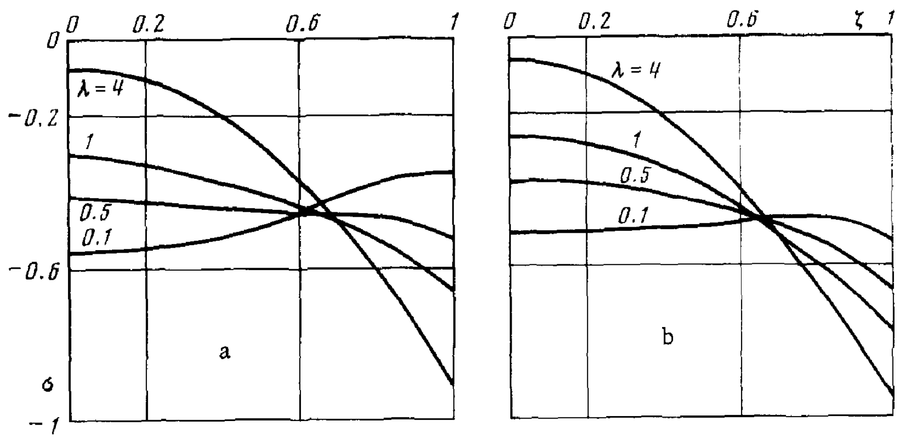

Fig 2

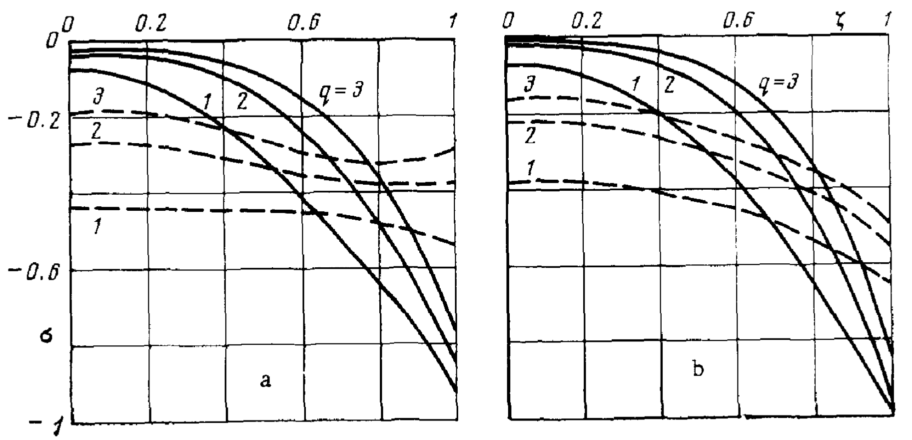

Fig 3

Table 1.

p	$\mathrm{Re}\gamma_p$	$\mathrm{Im}\gamma_p$	$\mathrm{Re}\gamma_p$	$\mathrm{Im}\gamma_p$	p	$\mathrm{Re}\gamma_p$	$\mathrm{Im}\gamma_p$	$\mathrm{Re}\gamma_p$	$\mathrm{Im}\gamma_p$
1	1.539	1.024	1.363	1.155	6	12.830	5.763	10.917	8.171
2	3.814	1.970	3.274	2.552	7	15.084	6.712	12.828	9.576
3	6.068	2.918	5.185	3.956	8	17.338	7.660	14.739	10.981
4	8.322	3.867	7.096	5.361	9	19.592	8.609	16.650	12.386
5	10.576	4.815	9.007	6.766	10	21.846	9.557	18.560	13.791

The annular stresses were found using the formula

$$\frac{1}{P}\sigma_{\theta\theta} = -\frac{a}{r^2} + 2\mathrm{Re}\sum_p \left[s_p(\zeta) + \frac{1}{r} n_p(\zeta) P_0^-(\gamma_p{}^*r)\right] \alpha_p K_0(\gamma_p{}^*r) \qquad (2.1)$$

The figures show the values of the annular stresses $\sigma = (\sigma_{\theta\theta} / P)_\Omega$, calculated according to the formula (2.1) near the lateral surface. Fig. 1 shows the case when the load $g(\zeta) = \zeta^2$ and $\lambda = 0.5$). The dashed line refers to an isotropic material, *1* to cadmium, *2* to cadmium with elastic constants $\nu = 0.07, \nu_z = 0.26, \nu_2 = 0.92, s_0^2 = 1.93 / 1.85$, used in [3], *3* to zinc and *4* to zinc with elastic constants $\nu = -0.058$, $\nu_z = 0.257$, $\nu_2 = 0.869$, $s_0^2 = 1.655$ taken from [6].
Figure 2 depicts the curves for the same stresses under the load $g(\zeta) = \zeta^2$ for varying relative thicknesses of the cadmium (a) and (b) plates; Fig. 3 gives the annular stresses under varying loads for cadmium (a) and for zinc (b), with the solid lines corresponding to the relative thickness $\lambda = 4$, and the dashed lines to $\lambda = 0.5$.

Figures 2 and 3 show that the stress distribution over the thickness is very nonuniform, therefore the conclusions concerning the errors of the empirical theory are similar to those made in [2] for the isotropic plates.

The value of the coefficients a and α_p obtained were used to check how the boundary conditions were satisfied on the lateral surface. The deviation of the values σ_{rr} obtained from those specified does not exceed 10^{-3}, and the stresses $\sigma_{r\zeta}$ at the boundary are of the order of 10^{-4}—10^{-3}.

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